Number Theory: Session 1
Stanford Pre-Collegiate Summer Institutes 2018

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Course Description: Number Theory is the study of the integers and their properties. For thousands of years, the greatest minds have been working to produce what many consider to be some of the most elegant and powerful ideas in all of mathematics. Number Theory continues to be an area of active research, and with the increasing power and availability of computers, there have been significant developments in applications of number theory that would not have been possible even 50 years ago. Our overarching goal in this course will be to observe, investigate, conjecture, and prove the patterns and relationships we see occurring in the integers.

Course Objectives: By the end of this course, students should gain a deeper understanding of number theory and gain valuable experience writing proofs, presenting solutions to mathematical problems, and communicating clearly with peers about mathematical concepts.

Course Format: Each day will consist of a combination of lecture, examples, student presentations, discussions of problem-solving strategy and techniques, and individual and group work on exercises and problem sets. Work on these problem sets will continue during the afternoon problem sessions. During our class time, students should be prepared to present solutions to exercises from the previous day's problem sets, as well as participate in the current day's activities.

Afternoon Session Format: After each morning class session, there will be a problem set assigned which is relevant to that day's activities. Students are **not** expected to complete every single problem from each problem set, though it will be advantageous to the student to *attempt* as many problems as time and interest permits. Students are encouraged to solve problems that they find interesting and challenging. A group of teaching assistants (TA's) will be on hand to help students with problem-solving and proof techniques, and to answer any questions that may arise when working on the problem sets.

Near the end of each afternoon problem session, students should pick one to three problems from the given problem set and write up complete solutions or proofs to be turned in. The student should focus on writing neat, clear, mathematically sound proofs. Use complete sentences, and organize the proof so that each idea flows logically from one step to the next. Write on separate pieces of paper and make sure that the statement of the problem is clearly stated. The proof should be written in such a way that a fellow mathematician could read through your solution and understand your argument without difficulty. Be proud of the work that you turn in!

By the end of each afternoon problem session, a few students should volunteer to present their solution to their peers during the next day's class session. When writing up solutions, students should start thinking about which problems they might like to present to the class the following day.

Final Presentations: In the first few days of the course, small groups of 3-4 students will be formed. Students are *strongly* encouraged to work with the other members of their group (and with other groups) during the afternoon problem sessions. Adjustments to these groups can be made as necessary.

At end of the last week of the course, these small groups will give a 25-30 minute presentation on an advanced number theory topic that is either a topic we did not cover in class, or is an extension of a topic we did cover in class. These presentations should be creative and interactive! Students are encouraged to include exercises, worksheets, games, posters, and anything else relevant to the topic that will be engaging to the class. Students should expect to choose a topic by the end of the first week and submit a presentation outline and summary by the end of the second week.

Course Schedule: This is a tentative course schedule, and changes will be made daily based upon the background and interests of the class.

Class	Date	Topic
1	June 26	-Ch. 0: The Four Numbers Game -Ch. 1: The Basics: Notation and Terminology -Ch. 2: Elementary Properties of Divisibility
2	June 27	-Ch. 3: Proof by Contradiction -Ch. 4: Mathematical Induction
3	June 28	-Ch. 5: The Greatest Common Divisor -Ch. 6: Prime Factorization and the Fundamental Theorem of Arithmetic
4	June 29	-Ch. 7: Introduction to Modular Arithmetic and Congruences -Ch. 8: Applications of Modular Arithmetic and Congruences -Movie: Hard Problems: The Road to the World's Hardest Math Contest -Work on selecting group presentation topics (Afternoon Session)
5	July 2	**Group Presentation Topic Selection Due** -Ch. 9: Linear Congruence Equations (problem solving with linear congruences) -Ch. 10: Fermat's Little Theorem
6	July 3	-Ch. 11: The Euler Phi-Function and the Euler-Fermat Theorem -Ch. 13: Squares Modulo p and Quadratic Residues (first half) -Ch. 24: Pythagorean Triples (Afternoon Session)
7	July 4	-Ch. 13: Squares Modulo p and Quadratic Residues (second half) -Ch. 14: Introduction to Quadratic Reciprocity -Ch. 25: Which Primes are Sums of Two Squares? (Afternoon Session)
8	July 5	-Ch. 15: The Law of Quadratic Reciprocity -Ch. 17: Diophantine Equations -Ch. 28: Square-Triangular Numbers and Pell's Equation -Work on group presentation outlines (Afternoon Session)
9	July 6	**Outline and Summary for Group Presentation Due** -Ch. 29: The Equation $x^4 + y^4 = z^4$ -Ch. 43: The Card Game SET -A Primer on Modular Forms -Movie: BBC Horizon - Fermat's Last Theorem
10	July 9	-Ch. 30: Cubic Curves and Elliptic Curves -Ch. 32: Elliptic Curves with Few Rational Points (first half)
11	July 10	-Ch. 32: Elliptic Curves with Few Rational Points (second half) -Ch. 33: Points on Elliptic Curves Modulo p
12	July 11	-Group Presentations -Ch. 34: Torsion Collections Modulo p and Bad Primes
13	July 12	-Group Presentations -Ch. 35: Defect Bounds and Modularity Patterns -Ch. 36: Elliptic Curves and Fermat's Last Theorem
14	July 13	-Group Presentations -Ch. 37: What's New in Number Theory? -SET Tournament (Afternoon Session) -Course Summary/Wrap-up